

12th Grade Math & Science Summer Packet

SIMPLIFYING RADICALS:

WE SAY THAT A SQUARE ROOT RADICAL is **simplified**, or in its **simplest form**, when the **radicand** has no **square** factors.

A radical is also in simplest form when the radicand is not a fraction.

Example 1. 33, for example, has no square factors. Its factors are $3 \cdot 11$, neither of which is a square number. Therefore, $\sqrt{33}$ is in its simplest form.

Example 2. Extracting the square root. 18 has the square factor 9.

$$18 = 9 \cdot 2.$$

Therefore, $\sqrt{18}$ is not in its simplest form. We have,

$$\sqrt{18} = \sqrt{9 \cdot 2}$$

We may now extract, or take out, the square root of 9:

$$\sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}.$$

$\sqrt{18}$ is now simplified. The radicand no longer has any square factors.

The justification for taking out the square root of 9, is this theorem:

$$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$$

*The square root of a product
is equal to the product of the square roots
of each factor.*

(We will prove that when we come to rational exponents, Lesson 29.

Here is a simple illustration: $\sqrt{100} = \sqrt{4 \cdot 25} = \sqrt{4} \cdot \sqrt{25} = 2 \cdot 5 = 10.$)

As for $\sqrt{9 \cdot 2}$, then, it is equal to the square root of 9 times the square root of 2, which is irrational. $3\sqrt{2}$.

Example 3 Simplify $\sqrt{75}$.

Solution. $\sqrt{75} = \sqrt{25 \cdot 3} = 5\sqrt{3}$.

75 has the square factor 25. And the square root of 25 times 3 is equal to the square root of 25 times the square root of 3.

$\sqrt{75}$ is now simplified.

Example 4. Simplify $\sqrt{42}$.

Solution. We have to factor 42 and see if it has any square factors. We can begin the factoring in any way. For example,

$$42 = 6 \cdot 7$$

We can continue to factor 6 as $2 \cdot 3$, but we cannot continue to factor 7, because 7 is a [prime number](#) (Lesson 32 of Arithmetic). Therefore,

$$42 = 2 \cdot 3 \cdot 7$$

We now see that 42 has no square factors -- because no factor is repeated. Compare Example 1 and Problem 2 of the [previous Lesson](#).

$\sqrt{42}$ therefore is in its simplest form.

Example 5. Simplify $\sqrt{180}$.

Solution. We must look for square factors, which will be factors that are repeated.

$$180 = 2 \cdot 90 = 2 \cdot 2 \cdot 45 = 2 \cdot 2 \cdot 9 \cdot 5 = 2 \cdot 2 \cdot 3 \cdot 3 \cdot 5$$

Therefore,

$$\sqrt{180} = 2 \cdot 3\sqrt{5} = 6\sqrt{5}.$$

Simplify. Use absolute value signs when necessary.

1) $\sqrt{24}$

2) $\sqrt[3]{1000}$

3) $\sqrt[3]{-162}$

4) $\sqrt{512}$

5) $\sqrt[4]{128n^8}$

6) $\sqrt{98k}$

7) $\sqrt[5]{224r^7}$

8) $\sqrt[3]{24m^3}$

9) $\sqrt{392x^2}$

10) $\sqrt{512x^2}$

11) $\sqrt[4]{405x^3y^2}$

12) $\sqrt[3]{-16a^3b^8}$

13) $\sqrt[4]{128x^7y^7}$

14) $\sqrt[3]{16xy}$

RATIONAL EXPRESSIONS:

A **rational expression** is something of the form

$$\frac{\text{polynomial}}{\text{polynomial}}.$$

Rational expressions can often be simplified by factoring the top and bottom, then cancelling any common factors. However, cancelling common factors can change the **domain** of the expression, and in some cases, this is important. In many of the examples below, I'll just do the cancellation without comment --- but first, here are some examples which show what the issue is.

Example. The following cancellation is valid for $x \neq 1$:

$$\frac{x^2 - 1}{x - 1} = \frac{(x - 1)(x + 1)}{x - 1} = x + 1.$$

If $x = 1$, then the common factor $x - 1$ is equal to 0, so in that case it is like simplifying $\frac{0}{0}$ to get 1 (which isn't right).

Notice that the original expression $\frac{x^2 - 1}{x - 1}$ is undefined if $x = 1$ (since $x = 1$ makes the bottom 0), but the final expression $x + 1$ is defined when $x = 1$ (it's equal to 2).□

Example. The following cancellation is valid for $x \neq -2$:

$$\frac{x^2 + 4x + 4}{x^2 + 5x + 6} = \frac{(x + 2)^2}{(x + 2)(x + 3)} = \frac{x + 2}{x + 3}.$$

When $x = -2$, the common factor $x + 2$ is equal to 0.□

Example. The following cancellation is valid for $x \neq -1$:

$$\frac{x^2 - 7x - 8}{x^2 + 3x + 2} = \frac{(x - 8)(x + 1)}{(x + 2)(x + 1)} = \frac{x - 8}{x + 2}.$$

When $x = -1$, the common factor $x + 1$ is equal to 0.□

Example. The following cancellation is valid for $x \neq 0$:

$$\frac{x^4 - x^2}{x^3 + 5x} = \frac{x^2(x^2 - 1)}{x^2(x + 5)} = \frac{x^2 - 1}{x^2 + 5}.$$

When $x = 0$, the common factor x^2 is equal to 0.□

State the excluded values for each.

$$1) \frac{60x^3}{12x}$$

$$2) \frac{70v^2}{100v}$$

$$3) \frac{m+7}{m^2+4m-21}$$

$$4) \frac{n^2+6n+5}{n+1}$$

$$5) \frac{35x-35}{25x-40}$$

$$6) \frac{-n^2+16n-63}{n^2-2n-35}$$

Simplify each and state the excluded values.

$$7) \frac{p+4}{p^2+6p+8}$$

$$8) \frac{9}{15a-15}$$

$$9) \frac{2a^2+10a}{3a^2+15a}$$

$$10) \frac{p^2-3p-10}{p^2+p-2}$$

$$11) \frac{x^2+x-6}{x^2+8x+15}$$

$$12) \frac{a^2+5a+4}{a^2+9a+20}$$

SCIENTIFIC NOTATION:

What you should be able to do with scientific notation:

- 1) Write/adjust any number to scientific notation
- 2) Know how to multiply and divide in scientific notation without using the exponent functions on your calculator
- 3) Use your calculator to perform any calculation by properly entering numbers in scientific notation.

General note on scientific notation. A number expressed in scientific notation has two parts:

- ** part 1: a coefficient (the part that looks like a simple number)
- part 2: a power of 10 (which contains an exponent)

Example: 3.5×10^4 . 3.5 is the coefficient. 10^4 is the power of 10. 4 is the exponent.

**** The coefficient must be greater than or equal to 1, but less than 10.**

Changing simple numbers to scientific notation:

The goal here is split your number into two parts... one bigger than the original by some power of 10 and one smaller than the original by the same power of 10. The changes to the power of 10 will cancel out leaving a number with the same value, but a different form. This is accomplished by moving the decimal of the number then multiplying by the correct power of 10.

step 1) Making sure you count places, move the decimal until the coefficient is between 1 and 10. This will make the number bigger or smaller by some power of 10.

step 2) Make the exponent the "opposite" of what you did to the coefficient.

- if the decimal was moved to the right (bigger), the exponent must be NEGATIVE to make the number smaller again.
- if the decimal was moved to the left (smaller), the exponent must be POSITIVE to make the number bigger again.

Example: write 0.00042 in scientific notation

step 1) move decimal 4 places to the right (0.00042 --> 4.2). (4 powers of 10 bigger)

step 2) the exponent must be -4 (4 powers of 10 smaller). Correct scientific expression: 4.2×10^{-4}

Example: write 3567 in scientific notation

step 1) move decimal 3 places to the left (3567 --> 3.567). (3 powers of 10 smaller).

step 2) the exponent must be 3 (3 powers of 10 bigger). Correct scientific expression: 3.567×10^3

Multiplying:

step 1) multiply the coefficients

step 2) multiply the powers of 10. **Hint:** to do this add the exponents to get a new exponent

step 3) combine new coefficient and exponent and adjust the answer to proper scientific notation

Example: $(4 \times 10^{-2})(3 \times 10^5)$

step 1) $4 \times 3 = 12$; step 2) $(10^{-2})(10^5) = 10^{-2+5} = 10^3$; step 3) $12 \times 10^3 = 1.2 \times 10^4$

Numbers do not have to be in proper scientific notation before you multiply them! You can (and should) always adjust them to proper notation at the end. Here are two common examples

Example: A coefficient greater than 10: $(40 \times 10^{-2})(3 \times 10^5)$

step 1) $40 \times 3 = 120$; step 2) $(10^{-2})(10^5) = 10^{-2+5} = 10^3$; step 3) $120 \times 10^3 = 1.2 \times 10^5$

Example: Only one number has an exponent $2.5 * (6 \times 10^{23})$ Write the other number with an exponent of zero

Since $2.5 = 2.5 \times 10^0$, you can turn the problem into “ $(2.5 \times 10^0) * (6 \times 10^{23})$ ” and do as shown below.

$$\text{step 1) } 2.5 \times 6 = 15; \quad \text{step 2) } (10^0)(10^{23}) = 10^{0+23} = 10^{23}; \quad \text{step 3) } 15 \times 10^{23} = 1.5 \times 10^{24}$$

Dividing:

Step 1) Divide the coefficients

Step 2) Divide the powers of 10. **Note:** to do this subtract the exponents to get a new exponent

Step 3) Combine new coefficient and exponent and adjust the answer to proper scientific notation

Example: $\frac{2 \times 10^4}{8 \times 10^7} = ?$

$$\text{Step 1) } \frac{2}{8} = 0.25; \quad \text{Step 2) } \frac{10^4}{10^7} = 10^{4-7} = 10^{-3}; \quad \text{Step 3) } 0.25 \times 10^{-3} = 2.5 \times 10^{-4}$$

Example: $\frac{88 \times 10^4}{8 \times 10^{-3}} = ?$ (again... be careful with negative exponents)

$$\text{step 1) } \frac{88}{8} = 11; \quad \text{step 2) } \frac{10^4}{10^{-3}} = 10^{4-(-3)} = 10^7; \quad \text{step 3) } 11 \times 10^7 = 1.1 \times 10^8$$

Just like when multiplying, numbers do not have to be in proper scientific notation before you divide them! Fix them at the end. The following examples are very similar to the multiplying ones:

Example: A coefficient greater than 10: $\frac{100 \times 10^4}{6 \times 10^7} = ?$

step 1) $\frac{100}{6} = 16.6$; step 2) $\frac{10^4}{10^7} = 10^{4-7} = 10^{-3}$; step 3) $16.6 \times 10^{-3} = 16.6 \times 10^{-2}$

Example: Only one number has an exponent $2.5 \div (6 \times 10^{23})$ Write the other number with an exponent of zero

Since $3.7 = 2.5 \times 10^0$, you can turn the problem into “ $(3.7 \times 10^0) \div (6 \times 10^{23})$ ” and do as shown below.

step 1) $3.7 \div 6 = 0.617$; step 2) $(10^0) \div (10^{23}) = 10^{0-23} = 10^{-23}$; step 3) $0.617 \times 10^{-23} = 4.17 \times 10^{-24}$

Adding or Subtracting:

Note: Numbers can only be added or subtracted in scientific notation if they **have the same exponent!** If they do not have the same exponent, one must be rewritten:

When changing the exponents, if you need to make your exponent bigger, move the decimal to the left by adding +1 each time until you get to the desired exponent. If you need to make the exponent smaller, move the decimal point to the right by adding -1 each time until you get to the desired exponent.

Note: adjust the numbers so the exponents match (often it is best to use the larger one)

step 1) add or subtract the coefficients

step 2) keep the same exponent

step 3) combine new coefficient with exponent and adjust the answer to proper scientific notation

Example: $8 \times 10^5 + 3 \times 10^5$

step 1) $8 + 3 = 11$; step 2) power of ten = 10^5 ; step 3) $11 \times 10^5 = 1.1 \times 10^6$

Example: $8.000 \times 10^5 - 3 \times 10^2 \rightarrow$ step 0) $8.000 \times 10^5 - 0.003 \times 10^5$

step 1) $8.000 - 0.003 = 7.997$; step 2) power of ten = 10^5 ; step 3) 7.997×10^5

Practice! Do **NOT** use a calculator to do the exponents! You may use a calculator to multiply or divide the number parts. Give all answers in proper scientific notation. Use significant figures when they are learned! SHOW work when you need to adjust values to perform the operations! example 1) $4.8 \times 10^{-8} \times 2.8 \times 10^{12} = 13.44$ (coefficients done on calculator) $\times 10^4$ (exponents done in head) = 1.3×10^5 (adjusted in head) example 2) $3.4 \times 10^6 + 4 \times 10^5 = 3.4 \times 10^6 + 0.4 \times 10^6$ (adjustment of second term done in head) = 3.8×10^6 (calculation done in head)

1a. $5670 =$ _____

1b. $290000 =$ _____

2a. $0.00671 =$ _____

2b. $34 \times 10^6 =$ _____

3a. $451 \times 10^{-4} =$ _____

3b. $312 \times 10^3 =$ _____

4a. $0.9871 \times 10^7 =$ _____

4b. $0.054 \times 10^{-2} =$ _____

5. $(2.4 \times 10^7) \times (5.0 \times 10^3) =$

6. $(2.9 \times 10^{23}) \div (3.0 \times 10^{-2}) =$

7. $(21) \div (4.67 \times 10^7) =$

8. $(4.5 \times 10^{-8}) \times (6010) =$

9. $(13 \times 10^{-9}) \div (3.0 \times 10^{23}) =$

10. $(601 \times 10^{-20}) \times (0.022) =$

11. $(2.8 \times 10^{-2}) \times (641 \times 10^{-3}) =$

12. $(0.98 \times 10^7) + (3.41 \times 10^7) =$