Pre-Calculus

Summer Assignment 2018

The following packet contains topics and definitions that you will be required to know in order to succeed in Pre-Calculus this year. You are advised to be familiar with each of the concepts and to complete the included problems by Thursday, September 6, 2018. All of these topics were discussed in either Algebra II or Geometry and will be used frequently throughout the year. All problems that you are to complete are marked in bold. All problems are <u>expected</u> to be completed.

Section 1: Coordinates and Planes

Midpoints:

The midpoint of the interval with endpoints a and b is found by taking the average of the endpoints.

$$M = \frac{a+b}{2}$$

The midpoint of a segment with endpoints at (x_1, y_1) and (x_2, y_2) is found by taking the average of the two coordinate values.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Find the midpoint M of the segment with endpoints C (4, 4) and D (-2, 6).

Distance:

The distance d between any two points x_1 and x_2 on a real number line is:

$d = |x_{1-}\, x_2| = \sqrt{(x_2 - x_1)^2}$

The distance d between any two points (x_1, y_1) and (x_2, y_2) on a Cartesian plane is:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Find the distance between each pair of points:

(6, -2) and (-5, 12) **(5.8, -1) and (7, -4)**

Section 2: Lines

Lines:

Slope: $\frac{y_2 - y_1}{x_2 - x_1}$

Slope Intercept Form: y = mx + b

Standard Form: ax + by = c

Point-Slope Form: $y - y_1 = m(x - x_1)$

Intercepts:

In order to find the x-intercept(s) of an equation, you have to set y equal to zero and solve the equation for x. In order to find the y-intercepts of an equation, you have to set x equal to zero and solve the equation for y.

Parallel Lines:

Two lines whose graphs have the same slope.

Perpendicular Lines:

Two lines whose graphs have opposite reciprocal slopes.

Find the slope of the lines passing through each set of points:

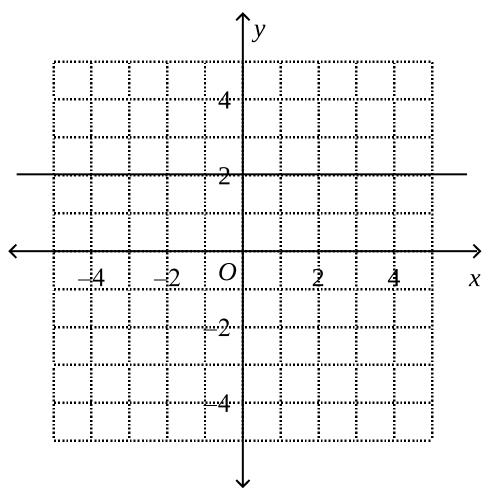
$(\mathbf{6}, \mathbf{12})\ \boldsymbol{and}\ (-\mathbf{6}, -\mathbf{2})$

$\left(-\frac{1}{3}, 0\right)\ \boldsymbol{and}\ \left(-\frac{1}{2}, -\frac{1}{2}\right)$

Find the slope of the line:

$\boldsymbol{3x + 5y = -15}$

Find the slope of the following Line:



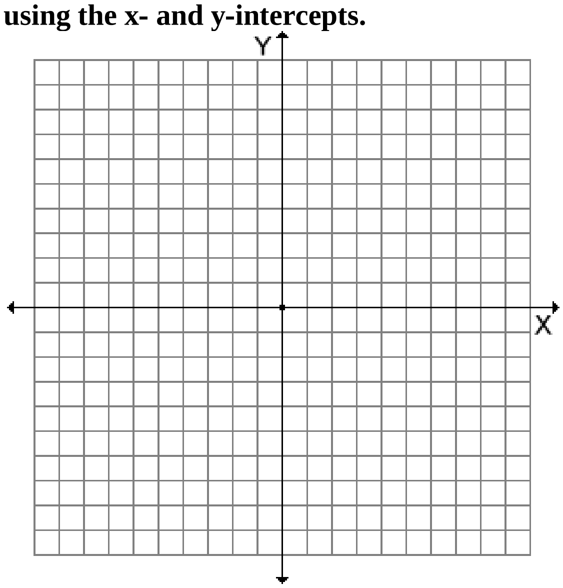
Write the equation of the line in standard form for a slope of -8 and through (-2, -2)

Find the point-slope form of the line through (-6, -4) and (2, -5).

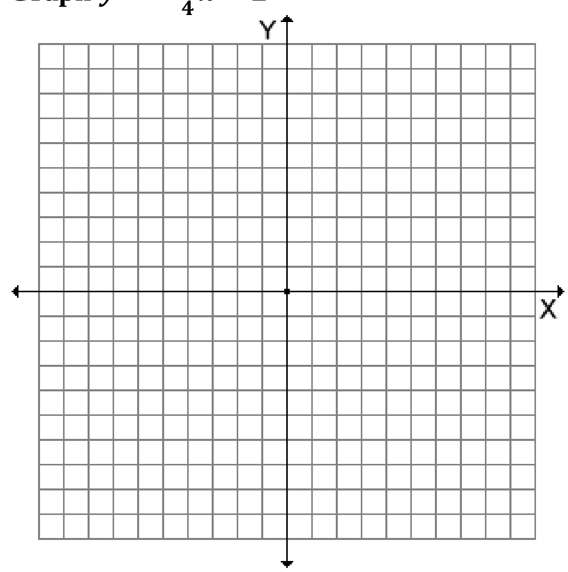
Write the equation of the line in slope-intercept form through $(-10, -3)$ and $(-1, 1)$.

Write the equation of the line of the horizontal line through $(-3, -1)$.

Graph the equation of 6x + 5y = 30 using the x- and y-intercepts.



Graph $y = -\frac{3}{4}x - 2$



Find the equation of the line that is parallel to $y = -3x + 4$ and goes through $(-4, 6)$.

Find the equation of the line perpendicular to $y = -\frac{5}{4}x + 1$ and goes through (2, 6).

Give the slope-intercept form for the equation of the line that is perpendicular to $7x + 3y = 18$ and contains $(6, 8)$.

Which two lines are parallel?

I. $5y = -3x - 5$

II. $5y = -1 - 3x$

III. $3y - 2x = -1$

What must be true about the slopes of two perpendicular lines, neither of which is vertical?

<u>Section 3: Functions</u>

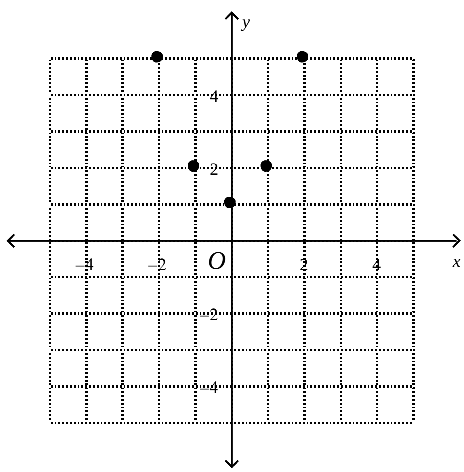
Function:

A function is a relation in which each value for the independent variable corresponds to a unique value of the dependent variable. A vertical-line test can be used on the graph of a relation in order to determine if the relation is a function. If a vertical-line can be drawn on the graph of a relation so that the line intersects 2 points on the graph, then the relation is not a function.

Domain: A list of all possible values for the independent variable.

Range: A list of all possible values for the dependent variable.

Write the ordered pairs for the relation. Find the domain and range and determine whether the relation is a function.



Suppose $f(x) = 4x - 2$ and $g(x) = -2x + 1$. Find the value of $\frac{f(5)}{g(-3)}$.

Evaluate each of the following for the function

$f(x) = x^2 - 4x + 7$**:**

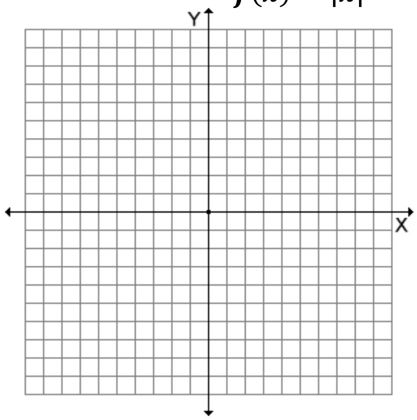
$f(4) =$

$f(-3) =$

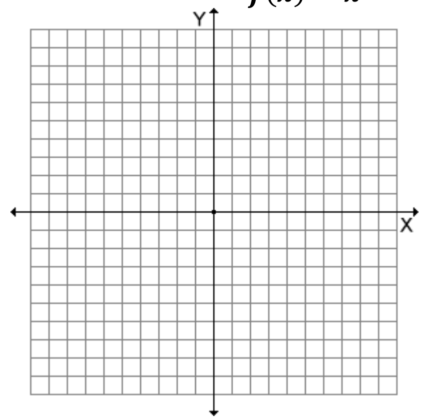
Given that $f(x) = x^2 - 4$ *and* $g(x) = x + 2$	**Show the work to find:** $f(g(x)) =$
	Show the work to find: $g(f(x)) =$

Sketch the graph of each functions:

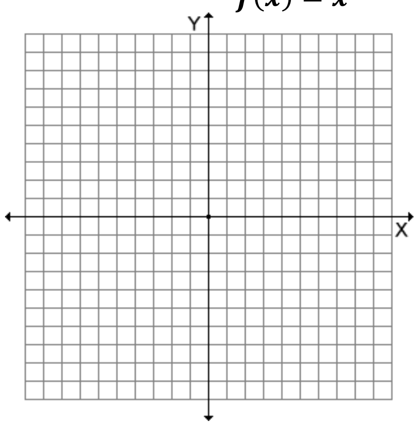
$f(x) = |x|$



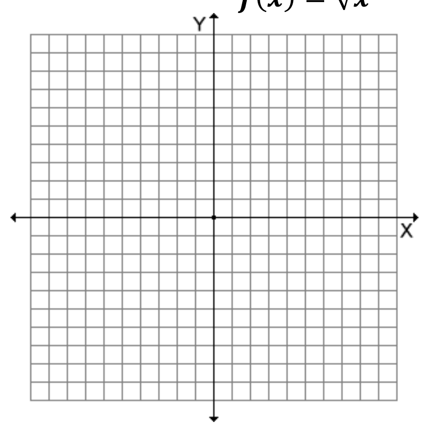
$f(x) = x$



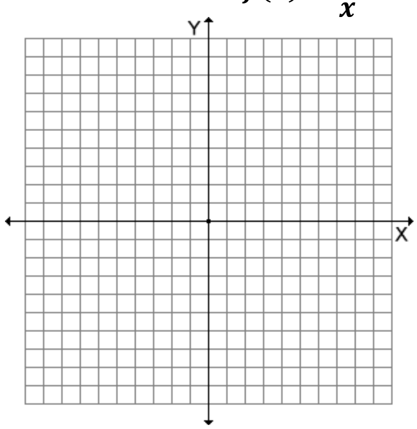
$f(x) = x^2$



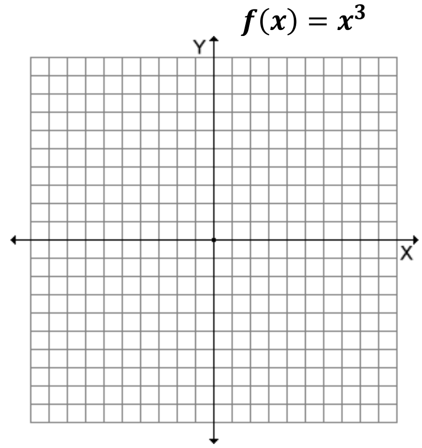
$f(x) = \sqrt{x}$



$f(x) = \frac{1}{x}$



$f(x) = x^3$



Section 4: Polynomials

Properties of Exponents:

1. Whole number exponents: $x^n = x \cdot x \cdot x \cdot \ldots \cdot x$ (n factors of x)
2. Zero exponents: $x^0 = 1, x \neq 0$
3. Negative Exponents: $x^{-n} = \frac{1}{x^n}$
4. Radicals (principal nth root): $\sqrt[n]{x} = a \rightarrow x = a^n$
5. Rational exponents: $x^{1/n} = \sqrt[n]{x}$
6. Rational exponents: $x^{m/n} = \sqrt[n]{x^m}$

Operations with Exponents:

1. Multiplying like bases: $x^n x^m = x^{m+n}$
2. Dividing like bases: $\frac{x^m}{x^n} = x^{m-n}$
3. Removing parentheses: $(xy)^n = x^n y^n$ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$ $(x^n)^m = x^{nm}$

Simplify each of the following expressions:

$\mathbf{2a^2b^{-4} \cdot 4a^{-8}b^6}$ $\mathbf{\frac{8a^2b^{-2}}{4a^4c^{-5}}}$ $\mathbf{\left(\frac{3x^4}{y^{-2}}\right)^3}$

$\frac{2x^3 + 6x}{4x}$ $\frac{5x - 4}{5x}$ $\frac{3xy^3 + x^2y^2}{xy}$

Solve each of the following by Factoring if possible. If not, use the quadratic formula:

$\mathbf{3x^2 - 6x = 0}$ $\mathbf{x^2 - 13x + 42 = 0}$

$\mathbf{4x^2 + 20x - 12 = 0}$ $\mathbf{x^2 - 2x - 35 = 0}$

$\mathbf{9x^2 - 42x + 49 = 0}$ $\mathbf{4x^2 - 144 = 0}$

$\mathbf{3x^2 - 16x + 5 = 0}$ $\mathbf{x^2 + 9 = 0}$

Solve each of the following by using the Quadratic Formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\mathbf{x^2 + 2x + 4 = 0}$ $\mathbf{8x^2 - 10x - 3 = 0}$

Expand each of the following (hint: FOIL):

$(x+4)^2$

$(x-2)^3$

Divide the following expression by both polynomial long division and synthetic division.

$$(x^4 + 15x^3 - 77x^2 + 13x - 36) \div (x - 4)$$

<u>Section V: Right Triangle Trigonometry</u>

Special Right Triangles

45-45-90: In a 45-45-90 triangle, the side lengths have a ratio of $a : a : a\sqrt{2}$.

30-60-90: In a 30-60-90 triangle, the side lengths have a ratio of $a : a\sqrt{3} : 2a$

Trigonometry:

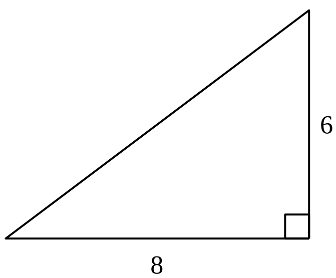
For any right triangle:

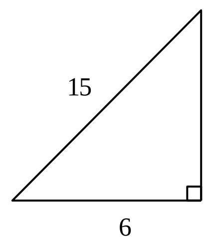
$$sin\theta = \frac{opposite\ side}{hypotenuse} \qquad cos\theta = \frac{adjacent\ side}{hypotenuse} \qquad tan\theta = \frac{opposite\ side}{adjacent\ side}$$

Pythagorean Theorem:

In a right triangle with legs a and b and hypotenuse c, then $a^2 + b^2 = c^2$.

Find the length of the missing sides of the figures. Leave your answer in simplest radical form.





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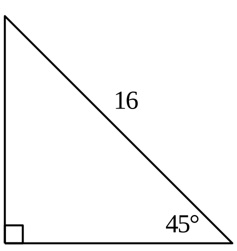

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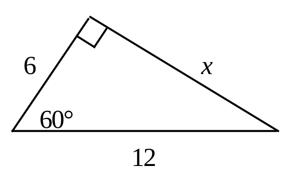

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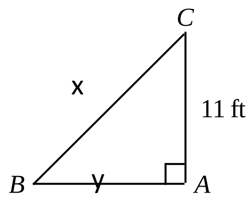
In triangle *ABC*, $\angle A$ is a right angle and $m\angle B$ = 45°. Find *BC*. If you answer is not an integer, leave it in simplest radical form.



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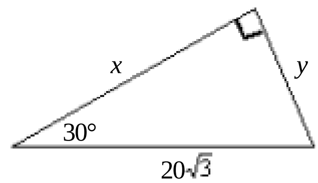

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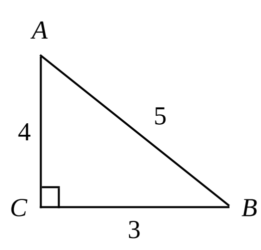
A piece of art is in the shape of an equilateral triangle with sides of 7 in. Find the area of the piece of art. Round your answer to the nearest tenth.

Write the ratios for sin *A*, cos *A*, *and* tan A.



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